Name:_____ Date:_____

HW Pre-Calculus 11 Section 5.3 Solving Equations with Radicals:

1. Solve each of the following equations algebraically. Make sure you check for extraneous roots:

a) $\sqrt{3x+7}=21$

$3x+7=21^2$

$3x=441-7$

$3x=334$

$x=\frac{334}{3}$

21 × 21 = 441

Check: $\sqrt{3\left(\frac{334}{3}\right)+7}=21$

$\sqrt{334+7}=21$

$\sqrt{441}=21$ ✓

b) $\sqrt{2x-5}-10=0$

$\sqrt{2x-5}=10$

$2x-5=100$

$2x=105$

$x=\frac{105}{2}$

Check: $\sqrt{2\left(\frac{105}{2}\right)-5}-10=0$

$\sqrt{105-5}-10=0$

$\sqrt{100}-10=0$

$10-10=0$ ✓

c) $5-\sqrt{2x-11}=3$

$5-3=\sqrt{2x-11}$

$2=\sqrt{2x-11}$

$4=2x-11$

$15=2x$

$\frac{15}{2}=x$

Check: $5-\sqrt{2\left(\frac{15}{2}\right)-11}=3$

$-\sqrt{15-11}=3-5$

$-\sqrt{4}=-2$

$-(2)=-2$ ✓

d) $2\sqrt{4x-1}+8=16$

$2\sqrt{4x-1}=16-8$

$\sqrt{4x-1}=4$

$4x-1=16$

$4x=17$

$x=\frac{17}{4}$

Check: $2\sqrt{4\left(\frac{17}{4}\right)-1}+8=16$

$2\sqrt{17-1}+8=16$

$2\sqrt{16}+8=16$

$2(4)+8=16$

$16=16$ ✓

e) $\sqrt{x}+2=x$

$\sqrt{x}=x-2$

$x=x^2-4x+4$

$0=x^2-5x+4$

$0=(x-4)(x-1)$

$x=4 \quad x=1$

Check $x=4$

$\sqrt{4}+2=4$

$2+2=4$ ✓

Check $x=1$

$\sqrt{1}+2=1$

$1+2\neq 1$

∴ $x=1$ is EXTRANEOUS

f) $4\sqrt{x}-4=x$

$4\sqrt{x}=x+4$

$16(x)=x^2+8x+16$

$0=x^2-8x+16$

$0=(x-4)(x-4)$

$x=4, \quad x=4$

Check

$4\sqrt{4}-4=4$

$4(2)-4=4$

$8-4=4$ ✓

g) $4-x=\sqrt{x^2-8}$

$(4-x)^2=x^2-8$

$16-8x+x^2=x^2-8$

$16+8=8x$

$24=8x$

$3=x$

Check

$4-3=\sqrt{3^2-8}$

$1=\sqrt{9-8}$

$1=\sqrt{1}$

$1=1$

h) $\sqrt{x^2+3}+x=3$

$\sqrt{x^2+3}=(3-x)$

$x^2+3=9-6x+x^2$

$3=9-6x$

$6x=6$

$x=1$

Check

$\sqrt{1^2+3}+1=3$

$\sqrt{4}+1=3$

$2+1=3$

$3=3$ ✓

i) $\sqrt{1+9x}+6=2x$

$\sqrt{1+9x}=2x-6$

$1+9x=(2x-6)^2$

$1+9x=4x^2-24x+36$

$0=4x^2-33x+35$

$4 \times -5 = -5$

$1 \times -7 = -28$

-33

$0=(4x-5)(x-7)$

$x=5/4$ EXTR

$x=7$

check $x=7$

$\sqrt{1+9(7)}+6=2(7)$

$\sqrt{64}+6=14$

$8+6=14$ ✓

check $x=5/4$

$\sqrt{1+9(\frac{5}{4})}+6=2(\frac{5}{4})$

$\sqrt{1+\frac{45}{4}}+6=\frac{10}{4}$

$\sqrt{\frac{49}{4}}+6=\frac{10}{4}$

$\frac{7}{2}+6 \neq \frac{10}{4}$

so $x=5/4$ is EXTR.

j) $\sqrt{7x^2-1}+1=3x$

$\sqrt{7x^2-1} \geq 3x-1$

$7x^2-1=(3x-1)^2$

$7x^2-1=9x^2-6x+1$

$0=2x^2-6x-2$

$0=x^2-3x-1$

CAN'T FACTOR, USE Q.F.

$a=1 \quad b=-3 \quad c=-1$

$x=\frac{3\pm\sqrt{9-4(1)(-1)}}{2}$

$=\frac{3\pm\sqrt{9+4}}{2}$

$x=\frac{3\pm\sqrt{13}}{2}$

$x=\frac{3+\sqrt{13}}{2}$ or $x=\frac{3-\sqrt{13}}{2}$ ← THIS ONE IS EXTR.

2. What is an extraneous root? How can you check if an equation has an extraneous root?

3. Which of the following equations will have an extraneous root? Explain, Indicate Yes or NO

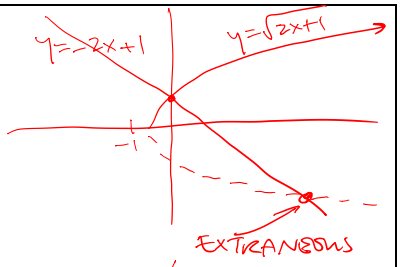
a) $\sqrt{3x+7}+10=0$

$\sqrt{3x+7}=-10$

EXTR. B/C the other side is negative!

b) $\sqrt{2x+1}=-2x+1$

SINCE THE LINE WILL HIT THE BOTTOM HALF OF THE ROOT FUNCTION, IT WILL HAVE AN EXTR. ROOT.



c) $-\sqrt{x}+9=0$

$-\sqrt{x}=-9$

$\sqrt{x}=9$

$x=81$

NO EXTR. ROOT

$3x+1=4x^2+24x+36$

$0=4x^2+21x+35$

$y(x)$

b^2-4ac

$21^2-4(4)(35)$

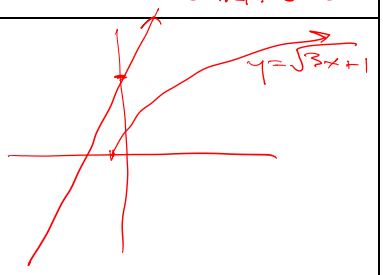
$441-560<0$

NO ROOTS

d) $\sqrt{3x+1}=2x+6$

THESE TWO GRAPHS WON'T INTERSECT!

THERE WON'T BE ANY ROOTS



4. Solve the following equations. Show all your work and steps:

a) $\sqrt{x+2}=\sqrt{2x+5}$ ① JUST SQUARE BOTH SIDES

$x+2=2x+5$

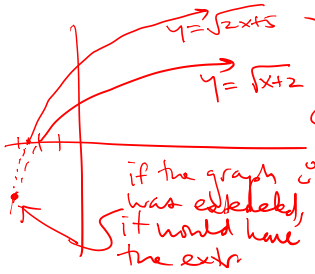
$2-5=2x-x$ ② check $x=-3$

$-3=x$

$\sqrt{-3+2}=\sqrt{2(-3)+5}$

$\sqrt{-1}=\sqrt{-1}$

↑ CAN'T SQ. ROOT NEG!



if the graph was extended, it would have the extr. ROOT.

$x=-3$ IS AN EXTR. ROOT.

b) $x-12=\sqrt{x}$

$(x-12)^2=x$

$x^2-24x+144=x$

$x^2-25x+144=0$

$(x-9)(x-16)=0$

$x=9, \quad x=16$

↑ EXTR!

check $x=9$

$9-12=\sqrt{9}$

$-3 \neq 3$

check $x=16$

$16-12=\sqrt{16}$

$4=4$ ✓

c) $x = 2\sqrt{x} - 1$

$x+1 = 2\sqrt{x}$

$(x+1)^2 = 4(x)$

$x^2 + 2x + 1 = 4x$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

Check $x=1$

$1 = 2\sqrt{1} - 1$

$1 = 2 - 1$

$1 = 1$ ✓

d) $x = 3 - \sqrt{x^2+3}$

$x - 3 = -\sqrt{x^2+3}$

$(x-3)^2 = (-1)^2(x^2+3)$

$x^2 - 6x + 9 = x^2 + 3$

$6 = 6x$

$1 = x$

Check

$1 = 3 - \sqrt{1+3}$

$1 = 3 - \sqrt{4}$

$1 = 3 - 2$

$1 = 1$ ✓

e) $\sqrt[3]{2x+3} = 4$

① CUBE BOTH SIDES

$2x + 3 = 4^3$

$2x + 3 = 64$

$2x = 61$

$x = \frac{61}{2}$

② check

$\sqrt[3]{2\left(\frac{61}{2}\right)+3} = 4$

$\sqrt[3]{61+3} = 4$

$\sqrt[3]{64} = 4$ ✓

f) $\sqrt[3]{(2x-1)^2} = 9$

① CUBE BOTH SIDES

$(2x-1)^2 = 729$

$4x^2 - 4x + 1 = 729$

$4x^2 - 4x - 728 = 0$

$x^2 - x - 182 = 0$

$(x-14)(x+13) = 0$

$x = 14 \quad x = -13$

② BOTH WORK

$182 = 91 \times 2$

$= 7 \times 13 \times 2$

g) $\sqrt{x+12} = 2 + \sqrt{x}$

$x + 12 = (2+\sqrt{x})(2+\sqrt{x})$

$x + 12 = 4 + 4\sqrt{x} + x$

$8 = 4\sqrt{x}$

$2 = \sqrt{x}$

$4 = x$

Check $x=4$

$\sqrt{4+12} = \sqrt{4} + 2$

$\sqrt{16} = 4$

$4 = 4$ ✓

h) $\sqrt{5x-1} + 3\sqrt{x} = 1$

$\sqrt{5x-1} = 1 - 3\sqrt{x}$

$5x - 1 = (1-3\sqrt{x})(1-3\sqrt{x})$

$5x - 1 = 1 - 6\sqrt{x} + 9x$

$6\sqrt{x} = 4x + 2$

$3\sqrt{x} = 2x + 1$

$9x = 4x^2 + 4x + 1$

$0 = 4x^2 - 5x + 1$

$0 = (4x-1)(x-1)$

$x = \frac{1}{4}, x = 1$

check. $x=1$

$\sqrt{5-1} + 3\sqrt{1} = 1$

$\sqrt{4} + 3 = 1$

$5 \neq 1$ EXTR

check $x = \frac{1}{4}$

$\sqrt{\frac{5}{4}-1} + 3\sqrt{\frac{1}{4}} = 1$

$\sqrt{\frac{1}{4}} + 3\left(\frac{1}{2}\right) = 1$

$\frac{1}{2} + 1.5 = 1$

$2 \neq 1$

extr

BOTH ARE EXTR

i) $\sqrt{2x+4} = 3 + \sqrt{x-5}$

$2x + 4 = (3+\sqrt{x-5})(3+\sqrt{x-5})$

$2x + 4 = 9 + 6\sqrt{x-5} + x - 5$

$x = 6\sqrt{x-5}$

$x^2 = 36(x-5)$

$x^2 = 36x - 180$

$x^2 - 36x + 180 = 0$

$(x-30)(x-6) = 0$

$x = 30, x = 6$

$180 = 18 \times 10$

$9 \quad 5$

$2 \quad 2$

$3 \quad 4$

$3 \quad 5$

$15 - 2 = 30$

$3 - 2 = 6$

Check $x=6$

$\sqrt{16} = 3 + \sqrt{1}$ ✓

Check $x=30$

$\sqrt{64} = 3 + \sqrt{25}$

$8 = 3 + 5$ ✓

BOTH ARE GOOD

j) $\sqrt{x} = \sqrt{x+4} - 1$

$\sqrt{x} + 1 = \sqrt{x+4}$

$(\sqrt{x}+1)(\sqrt{x}+1) = x+4$

$x + 2\sqrt{x} + 1 = x + 4$

$2\sqrt{x} = 3$

$4x = 9$

$x = \frac{9}{4}$

Check

$\sqrt{\frac{9}{4}} = \sqrt{\frac{9}{4}+4} - 1$

$\frac{3}{2} = \sqrt{\frac{25}{4}} - 1$

$\frac{3}{2} = \frac{5}{2} - 1$

$1.5 = 1.5$ ✓

k) $\sqrt{x+8}-\sqrt{x-7}=\sqrt{5}$

$\sqrt{5}+\sqrt{x-7}$

$x+8=(\sqrt{5}+\sqrt{x-7})(\sqrt{5}+\sqrt{x-7})$

$x+8=5+2\sqrt{5x-35}+x-7$

$10=2\sqrt{5x-35}$

$5=\sqrt{5x-35}$

$25=5x-35$

$60=5x$

$12=x$

Check

$\sqrt{20}-\sqrt{5}=\sqrt{5}$

$\sqrt{20}=2\sqrt{5}$

$\sqrt{20}=\sqrt{20}$ ✓

L) $\sqrt{3-x}-\sqrt{16+2x}=\sqrt{x+7}$

$2+\sqrt{x-5}=\sqrt{2x-3}$

$\frac{2}{\sqrt{x+1}}=\sqrt{x}+\sqrt{x+1}$

① FIND THE L.C.D. $\sqrt{x+1}$

② MULTIPLY ALL TERMS BY $\sqrt{x+1}$ TO CANCEL OUT THE DENOM.

$\frac{2}{\sqrt{x+1}}(\sqrt{x+1})=(\sqrt{x})(\sqrt{x+1})+(\sqrt{x+1})(\sqrt{x+1})$

$2=\sqrt{x^2+x}+x+1$

$1-x=\sqrt{x^2+x}$

$1-2x+x^2=x^2+x$

$1=3x$

$\frac{1}{3}=x$

Check

$\frac{2}{\sqrt{\frac{1}{3}+1}}=\sqrt{\frac{1}{3}}+\sqrt{\frac{1}{3}+1}$

$\frac{2}{\sqrt{4/3}}=\sqrt{\frac{1}{3}}+\sqrt{\frac{4}{3}}$

$1.732050808=1.732050808$ ✓

$\sqrt{4-x}+\sqrt{x-9}=\sqrt{x-14}$

$\frac{3}{\sqrt{x}}-5=\frac{1-2\sqrt{x}}{\sqrt{x}}$

① FIND THE L.C.D. $\sqrt{x}$

② CANCEL OUT DENOM. BY MULTIPLYING BY $\sqrt{x}$

$\left(\frac{3}{\sqrt{x}}\right)\sqrt{x}-5\sqrt{x}=\left(\frac{1-2\sqrt{x}}{\sqrt{x}}\right)(\sqrt{x})$

$3-5\sqrt{x}=1-2\sqrt{x}$

$2=3\sqrt{x}$

$4=9x$

$\frac{4}{9}=x$

③ check $x=4/9$

$\frac{3}{\sqrt{\frac{4}{9}}}-5=\frac{1-2\sqrt{\frac{4}{9}}}{\sqrt{\frac{4}{9}}}$

$\frac{3}{\left(\frac{2}{3}\right)}-5=\frac{1-2\left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right)}$

$4.5-5=\frac{1-\frac{4}{3}}{\frac{2}{3}}$

$-\frac{1}{2}=\frac{3-4}{2}$

$-\frac{1}{2}=-\frac{1}{2}$ ✓

5. Solve for "c" in terms of "a" and "b" given that: $\sqrt{a+\frac{b}{c}}=a\sqrt{\frac{b}{c}}$

$a+\frac{b}{c}=a^2\left(\frac{b}{c}\right)$

$a=\frac{a^2b}{c}-\frac{b}{c}$

$a=\frac{1}{c}(a^2b-b)$

$c=\frac{1}{a}(a^2b-b)$